The University of British Columbia

Midterm 1 - October 2, 2014

Mathematics 105

Section 101

Closed book examination Time: 50 minutes

Last Name _____ **First** _____ **Signature** _____

Student Number _____

Special Instructions:

No books, notes, or calculators are allowed.

Student Conduct during Examinations

• Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.

• Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.

• No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.

• Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.

• Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:

(a) speaking or communicating with other candidates, unless otherwise authorized;

(b) purposely exposing written papers to the view of other candidates or imaging devices;

(c) purposely viewing the written papers of other candidates;

(d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,

(e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)–(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).

• Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.

• Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.

• Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

1		15
2		5
3		10
4		10
5		10
Bonus		5
Total		50

1. (15 marks)
(a) (4 marks) Let:

$$f(x, y) = \sin(x^2 + xy) + e^y.$$

Find $\frac{\partial f}{\partial x}\,|_{(0,0)}$ and $\frac{\partial f}{\partial y}\,|_{(0,0)}$. *Simplify your answers.*

(b) (3 marks) Find all values of a such that $\mathbf{v} = \langle a, -2, -1\rangle$ is orthogonal to $\mathbf{w} = \langle a, 3, a\rangle$.

(c) (2 marks) Find an equation of the plane $\mathcal{P}$ which has the normal vector $\mathbf{n}_{\mathcal{P}} = \langle -1, 2, 0 \rangle$ and passes through the point $P(-3, 3, 5)$. *Simplify your answer.*

(d) (3 marks) Determine if the plane $\mathcal{P}$ in part (c) is parallel to the plane $\mathcal{Q}$ given by the equation $4x = 8y$. *Justify your answer.*

(e) (3 marks) Assume that $f(x, y)$ has continuous partial derivatives of all orders, and

$$f_{yx}(x, y) = e^{xy^2},$$

compute f_{xyy}. State in detail any result that you use.

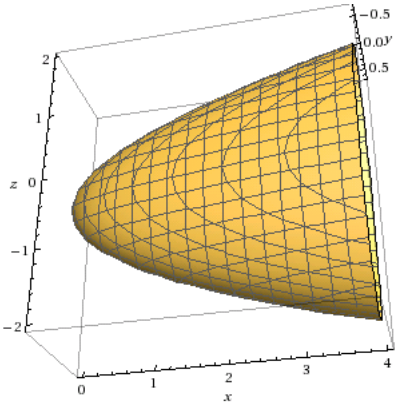
2. (5 marks) Consider the surface S given by:

$$z^2 = x^2 - 4y^2.$$

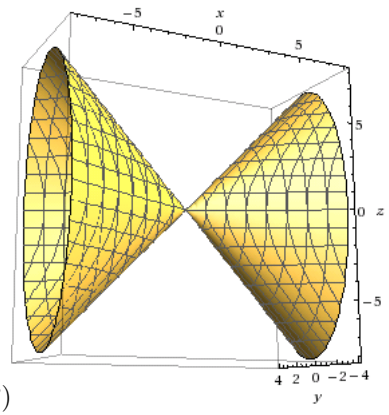
(a) (4 marks) Find and sketch the traces of S in the $x = 4$ and $z = 0$ planes. *Label clearly the axes, and all intercepts on the traces.*

(b) (1 mark) Based on the traces you sketched above, which of the following renderings represents the graph of the surface?

(A)



(B)



3. (10 marks) Let R be the region $\{x^2 + 4y^2 \leq 4, x \leq 0\}$. Find the maximum and minimum values of the function

$$f(x, y) = 6 - x^2 - 2y^2.$$

on the *boundary of the region R*.

4. (10 marks) Find *all* critical points of the following function:

$$f(x, y) = x^4 + y^4 - 4xy.$$

Classify each point as a local minimum, local maximum, or saddle point.

5. (10 marks) A company produces:

$$P(x, y) = 5x^{\frac{1}{5}} y^{\frac{4}{5}}$$

units of goods per week, utilizing x units of labour and y units of capital. If labour costs \$1 per unit, and capital costs \$1 per 8 units, use the method of Lagrange multiplier to find the most cost-efficient division of labour and capital that the company can adopt if its goal is to produce 80 units of goods per week.
Clearly state the objective function and the constraint. *You are not required to justify that the solution you obtained is the absolute maximum.* **A solution that does not use the method of Lagrange multipliers will receive no credit, even if it is correct.** *Note:* $32 = 2^5$.

Bonus. (5 marks)
Find and sketch the domain of the function $f(x, y) = \arcsin(x^2 + y^2 - 5)$. *Shade the area of the domain, and label all intercepts.*

The End